Quantum golden field theory – Ten theorems and various conjectures

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Abstract

Ten theorems and few conjectures related to quantum field theory as applied to high energy physics are presented. The work connects classical quantum field theory with the golden mean renormalization groups of non-linear dynamics and *E*-Infinity theory.

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1. Introduction

The present work is an attempt to wrap up various results obtained recently regarding exceptional Lie symmetry groups hierarchies in high energy physics [1,2]. In addition, recent insights into the nature of Newtonian quantum gravity will be tied up with another approach namely *E*-Infinity theory in a Machian "Denk" economical fashion [3–6].

2. Few theorems and some conjectures regarding quantum gravity and unification

The following theorems are exact but some of them await rigorous proof. They are stated in general form as to remain independent from the details of the constituent relations of the field. Subsequently, we quantify the theorems but lacking proof we then call them conjectures although we are almost sure that they are with a probability bordering on certainty correct [1–8].

Theorem 1. *The exact integer value of the inverse fine structure constant is one fourth of the dimension of the first exceptional Lie groups hierarchy involving the E-line and including the grand unifications groups* (SU(5) *and* SO(10)) *as well as the standard model* (SU(3), SU(2), U(1)).

By applying the quantum super-position we find:

$$\frac{1}{4}\sum_{i=1}^{8}|E_i| = \bar{\alpha}_0(\text{integer}) = 137$$

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Proof. By simply superimposing dimensions and noting that $SU(5) = E_4$, $SO(10) = E_5$, $A_1A_2 = SU(3)SU(2) = E_3SU(3) = E_2$ and $U(1) = E_1$, one finds that

$$|E_8| + |E_7| + |E_6| + |E_5| + |E_4| + |E_3| + |E_2| + |E_1| = 248 + 133 + 78 + 45 + 24 + 11 + 8 + 1 = 548$$

Consequently, we have

$$\frac{1}{4}\sum_{i=1}^{8}|E_1| = \left(\frac{1}{4}\right)(548) = 137$$

exactly as claimed. $\square$

Corollary. *The exact theoretical value of the inverse electromagnetic fine structure constant is equal to one fourth of the sum of the dimensions of the transfinite union of E_i where we have to make the following "radiative" transfinite corrections* [1–6]

$$E_8 \rightarrow 248 - (k^2/2)$$
$$E_7 \rightarrow 133 - (8k + \phi^{12})$$
$$E_6 \rightarrow 78 + (3k)$$
$$E_5 \rightarrow 24 + (2 + 2k)$$
$$E_3, E_1 \rightarrow 12 - 2\phi^4$$
$$E_2 \rightarrow 10$$
$$\sum_{1}^{8}|E_i| = 548 + (4)(k_0)$$

where $\phi = (\sqrt{5} - 1)/2$ *is the golden mean,* $k = \phi^3(1 - \phi^3) = 0.18033899$ *and* $k_o = \phi^5(1 - \phi^5) = 0.082039325$.

This is exactly 4 multiplied by $\bar{\alpha}_0$ where $\bar{\alpha}_0 = 137 + k_0 = 137.082039325$ is the exact theoretical value of the inverse electromagnetic fine structure constant [1–6]

The accurate experimental value found by projection is:

$$\bar{\alpha}_0(\text{experimental}) = \frac{\bar{\alpha}_0 - k_0}{\cos(\pi/\bar{\alpha}_0)} = \frac{137}{\cos(\pi/\bar{\alpha}_0)} = 137.03598$$

It should be noted that the preceding transfinite "radiation" corrections are not unique and slightly different corrections lead to exactly the same result.

Theorem 2. *The exact inverse coupling of the non-super symmetric and the super symmetric quantum gravity coupling is given by* [1]

$$\bar{\alpha}_u = \begin{bmatrix} \bar{\alpha}_g \\ \bar{\alpha}_{gs} \end{bmatrix} = (\bar{\alpha}_3 + \bar{\alpha}_4) + \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}(\bar{\alpha}_0)(\phi^3) = (10) + \delta_{1,2}(32 + 2k) = (10) + \begin{bmatrix} 32 + 2k \\ 16 + k \end{bmatrix} = \begin{bmatrix} 42 + 2k \\ 26 + k \end{bmatrix}$$

Proof. The present proof depends in this form on the conjecture that

$$\ln\frac{M_U}{M_Z} = 32 + 2k$$

where M_U is the grand unification magnetic monopole and M_Z is the electroweak Z scale. Furthermore, we know it is theoretically exact that $\bar{\alpha}_1$, $\bar{\alpha}_2$ and $\bar{\alpha}_S$ of *E*-Infinity theory are $\bar{\alpha}_1 = 60$, $\bar{\alpha}_2 = (1/2)\bar{\alpha}_1 = 30$ and $\bar{\alpha}_S = \bar{\alpha}_{S1} + \bar{\alpha}_{S2} + \bar{\alpha}_{S3} = 8 + 1 + 1 = 10$ where $\bar{\alpha}_{S1} + \bar{\alpha}_{S2} = \bar{\alpha}_3$ and $\bar{\alpha}_{S3} = \bar{\alpha}_4$. Consequently $\bar{\alpha}_0$ may be reconstructed as

$$\bar{\alpha}_0 = (1/\phi)\bar{\alpha}_1 + \bar{\alpha}_2 + \bar{\alpha}_3 + \bar{\alpha}_4 = (1/\phi)(60) + 30 + 9 + 1 = 137 + k_0$$

Thus the classical one loop renormalization of the standard quantum field theory may be written as

$$\bar{\alpha}_u = (\bar{\alpha}_3 + \bar{\alpha}_4) + \delta_i \ln\frac{M_U}{M_Z}$$

Using our conjecture that $\ln(M_U/M_Z) = 32 + 2k$, the proof of our theorem follows for $\delta_1 = 1$ and $\delta_2 = 1/2$ $\square$

Theorem 3. *The inverse super symmetric gravity coupling is exactly half the natural logarithm of the Planck scale to the electromagnetic scale* [1,4]

$$\bar{\alpha}_{gs} = \frac{1}{2} \ln\left(\frac{E(\text{planck})}{E(\text{electromagnetism})}\right)$$

Conjecture

$$\bar{\alpha}_{gs} \cong \frac{1}{2} \ln\left(\frac{M_{\text{p}}}{m_{\text{e}}}\right) = 1/2\left(\frac{\ln(10)^{19}}{10^{-3}}\right)$$

and therefore

$$\bar{\alpha}_{gs} = \frac{1}{2}(52 + 2k) = 26 + k$$

The preceding theorem and the conjecture are quite intuitive and bound to be correct for the following obvious reasons:

The Planck mass is the scale of quantum gravity unification. The electron scale on the other hand is the scale of low energy physics. The logarithmic scaling of the ratio is thus an obvious measure for the coupling between the two. The factor 1/2 is due to the doubling of the number of particles in the super symmetric scenario.

The final result is as expected in full agreement with all our previous calculations and in particular Theorem 2.

Theorem 4. *The non-super symmetric inverse grand unification coupling $\bar{\alpha}_g$ is equal to the natural logarithm of the ratio between the mass of the unification magnetic monopole and the mass of the electron* [3]

Conjecture

$$\bar{\alpha}_0 \simeq (\delta_1 = 1)\left(\ln\frac{(GrandU)}{m_{\text{e}}}\right) \simeq (1)(\ln(10^{15}/10^{-3})) \simeq 41.44$$

and therefore

$$\bar{\alpha}_0 = 42 + 2k = 42.236067$$

exactly as predicted by Theorem 2.

We mention on passing that for $\ln\frac{M_U}{M_Z}$ when $M_u = M_z = M_{\text{planck}}$, we have $\ln\frac{M_{\text{p}}}{M_{\text{p}}} = \ln 1 = 0$ and $\bar{\alpha}_4 = 1$. Therefore, $\alpha_3 = \alpha_s = \infty$. That is the phase transition called confinement when a quark turns into a Planck mass [4].

Theorem 5. *The exact inverse electromagnetic fine structure constant is given by*

$$\bar{\alpha}_u = (\bar{\alpha}_3 + \bar{\alpha}_4) + \delta_3 \ln\frac{M_U}{M_Z}$$

for the following transformation which respects transfinite corrections [1,3]

$$\bar{\alpha}_3 + \bar{\alpha}_u = 10 \Rightarrow 10 - (2 + 2k)$$
$$\delta_3 \Rightarrow 1 + 3 = 4$$
$$\ln\frac{M_U}{M_Z} \Rightarrow 32 + 2k$$
$$\bar{\alpha}_u \Rightarrow \bar{\alpha}_0$$

Thus

$$\bar{\alpha}_0 = [2(4 - k)] + (4)[32 + 2k] = 136 + 6k = 137 + k_0$$

It is instructive to note the following integer approximations. First setting $k_o = 0$, one finds $\bar{\alpha}_0 = 137$. Setting on the other hand $k = 0$, one finds $\bar{\alpha}_0 = 136$. Finally setting in our transformation $k = 0$, one finds

$$\alpha_u = 10 + (4)(32) = 10 + 128 \simeq 138$$

However, if we return to the approximate classical theory with $\bar{\alpha}_4 = 1$, then one finds

$$\bar{\alpha}_u = (8 + 11) + 128 = 137 = \bar{\alpha}_0$$

The last equation is identical to that found in the 2-Adic expansion of $\bar{\alpha}_0 = 137$ namely

$$\|137\|_2 = \|2^7 + 2^3 + 2^0\|_2 = \|128 + 8 + 1\|_2 = \|137\|_2 \equiv 1$$

Theorem 6. *For a 9 + 1 = 10 spacetime the number of independent components of the n = 9 Riemann tensor is exactly equal to the sum of the principal E-line of the exceptional Lie symmetry groups minus 8. Thus we have*

$$R^{(9)} = (9)^2(9^2 - 1)/12 = \sum_i^8 |E_i| - 8 = 548 - 8 = 540$$

Conjecture. *The missing 8 from $R^{(9)} = 540$ which would have made equal 548 represents the extra scaler Higgs field degrees of freedom.*

Theorem 7. *The number of instantons of the K3 Kähler n = 24 is equal to the square root of $\sum_1^8 |E_i|$ of Theorem 5 plus $|D_4| = 28$. That means:*

$$n = \sqrt{\sum_1^8 |E_i| + |D_4|} = \sqrt{548 + 28} = \sqrt{576} = 24$$

We note that the Euler characteristic of K3 is $\chi = n$.

Theorem 8. *The exact number of instantons of the E-Infinity Fuzzy K3 Kähler is equal to the square root of the total sum of the major exceptional Lie symmetry groups hierarchy involving all principal exceptional Lie groups and some simple groups as well as the unification groups, the standard model and the Higgs field degrees of freedom. Thus we have*

$$\bar{\alpha}_{gs} = \sqrt{\sum_i^{12} |E_i|} = \sqrt{685.4101965} = 26 + k = 26.18033898$$

where by the superposition principle, we have

$$\sqrt{\sum_1^{12} |E_1|} = 248 + 133 + 78 + 45 + 24 + 12 + 8 + 28 + 52 + 14 + 42 + 1 + 4$$

$$= |E_8| + |E_7| + |E_6| + SO(10) + SU(5) + |\overline{M_s}| + |H| + |D_4| + |F_4| + |G_2| + |E_{6(6)}| + |U(1)| + \Delta = 685 + \Delta$$

where Δ are the sums of the transfinite corrections $\Delta = 0.410196625$. It is most interesting that while $\sqrt{576} = 24$, one finds the exact integer value of $\bar{\alpha}_{gs}$ namely 26 from $\sqrt{676} = 26$ and $676/2 = 338 \simeq |SL(2,7)|_c = 338.88543824$.

This value corresponds to an idealized group hierarchy given by

$$248 + 128 + 78 + 52 + 42 + 42 + 26 + 26 + 16 + 10 + 8 = |E_8| + |E_{8(8)}| + |E_7| + |F_4| + |E_{6(6)}| + |E_{6(-26)}|$$
$$+ D^{(16)} + D^{(10)} + |H| = 676$$

Theorem 9. *Newton's gravity fine structure constant is given by the average dimension of the 12 principal groups of the major exceptional Lie hierarchy lifted to 12 dimensions bi-jectionally and squared. It corresponds to a Clifford–Finkelstein gravity manifold [5,6].*

Conjecture. *Based on the above theorem, we conjecture that*

$$\sqrt{\bar{\alpha}_{NG}} = \left(\frac{685.4161965}{12}\right)^{12-1} = (57.11751638)^{11} = (2.1108)(10)^{19}$$

We may note that Clifford–Finkelstein gravity manifold is dimensionally close to $D \cong (2)^{127} \cong (10)^{38} \simeq \bar{\alpha}_{NG} = [(10)^{19}]^2$

It is related to a remarkable map

$$n = 1 \rightarrow 2^n - 1 = 1 \text{ (limit cycle)}$$
$$n = 2 \rightarrow 2^2 - 1 = 3$$
$$n = 3 \rightarrow 2^3 - 1 = 7$$
$$n = 7 \rightarrow 2^7 - 1 = 127$$
$$n = 127 \rightarrow 2^{127} - 1 \simeq (1.7)(10)^{38}$$

From the above, we see that we have the following hierarchy (1, 5, 6)

$$1$$
$$1 + 3 = 4$$
$$4 + 7 = 11$$
$$11 + 127 = 138$$
$$138 + 2^{127} - 1 = 137 + 2^{127}$$

This is probably the only equation in physics which relates an enormous constant-like 2^{127} to such a, by comparison, minute coupling as $\bar{\alpha}_0 = 137$.

Theorem 10. *The exact grand unification coupling constant and Newton's gravity fine structure constant have an exact formal analogy.*

For $\bar{\alpha}_g$ we have:

$$\bar{\alpha}_g = \frac{|E_8 E_8|_c}{\left(|\mathbf{SM}|_c = \sqrt{\bar{\alpha}}\right)} = \frac{496 - k^2}{\left(12 - 2\phi^4 = \sqrt{\bar{\alpha}_0}\right)} = 42 + 2k$$

while for $\bar{\alpha}_{QG}$ we have $\sqrt{\bar{\alpha}_{QG}} = \frac{N(\text{planck})}{\bar{\alpha}_{QG}}$ where $\bar{\alpha}_{QG} = 1$ as well known from $R = \ln(\text{planck})$ of Kaluza–Klein theory and $N($Planck$)$ is given by

$$N(\text{Planck}) = \frac{M_{\text{planck}}}{M_{\text{protons}}} = \frac{(10)^{19}}{1} \simeq (10^{19})$$

Consequently,

$$\bar{\alpha}_{NG} \cong ((10)^{19})^2 \simeq (10)^{38}$$

3. Conclusion

We have presented ten theorems and few conjectures related to what may be termed golden mean field theory. The theorem connects for the first time two totally new aspects of average symmetries based on the sum of the dimension of exceptional Lie groups hierarchies. The results are quite startling and could be only explained by the fact that the micro cosmos possesses an incredibly high degree of chaotic symmetry akin to the melody of a large orchestra playing a master symphony. Piano, strings and musical instruments mix chaotically but the final result is a highly tuned magnificent work of art. I am sure that neither Goethe, Michelangelo, Beethoven, Heisenberg nor Dirac would have found that unexpectedly surprising.

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